$\alpha_s(m_Z)$, intermediate scales and Fermion masses in supersymmeric theories; two-loop results

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One way to generate an intermediate scale being consistent with gauge coupling unification is to add new Higgs scalars above the intermediate scale. We classify such scenarios according to their degree of departure from minimal supersymmetric standard model. Thereafter, we summarize the results of a two-loop renormalization group analysis of the gauge and the Yukawa sectors of such scenarios, and their sensitivity to the *input* $\alpha_3(m_Z)$. The presence of an adjoint color octet above the intermediate scale can raise the gauge coupling unification scale to the *string* scale, acommodate a suitable intermediate scale, provide a left-handed τ-neutrino of mass of the order of few electron volts and also have the right prediction of the low energy ratio of m_b/m_τ.

There are several physical arguments suggesting that in an unified theory like SO(10) there may be an intermediate scale [1] corresponding to a left-right gauge symmetry breaking [2] somewhere around 10^{11} to 10^{12} GeV based on neutrino physics [3,4] as well as strong CP problem [5,6]. In this talk we summarize a two-loop analysis [7] of a class of intermediate-scale supersymmetric SO(10) models [8–13] when the strong coupling constant $\alpha_3(m_Z)$ in the range 0.110 to 0.130 [14] [See Figure (1a)]. One-step unification *predicts* $\alpha_3(m_Z) \geq 0.126$ including the supersymmetric threshold corrections [15].

Equivalently, $\alpha_3(m_Z)$ can also be treated as an *input*. Three inputs in the gauge sector namely $\alpha_1(m_Z)$, $\alpha_2(m_Z)$ and $\alpha_3(M_Z)$ can determine the three unknown parameters M_X, M_I and $\alpha_G(M_X)$. Given $\alpha_1(m_Z)$ and $\alpha_2(m_Z)$, there exists a value of $\alpha_3(m_Z)$ for which $M_X = M_I$. In one loop approximation this unique value, for which $M_X = M_I$, is $\alpha_3(m_Z) \sim 0.1144$. Lee and Mohapatra [10] have studied a number of models where it is possible to get an intermediate scale $M_I \leq M_X$ if $\alpha_3 \neq 0.1144$ and if the scalar structure above M_I is enlarged.

The 1-loop renormalization group equation (RGE) of the three couplings introducing a general intermediate scale M_I between M_Z and M_X is our starting point. We have used b_i to denote the beta function coefficients below the intermediate scale and $b_i^\prime$ to denote them above the intermediate scale. The relations are,

$$\alpha_i^{-1}(m_Z) \quad = \quad \alpha_G^{-1} + \frac{b_i}{2\pi} \ln \frac{M_I}{M_Z} + \frac{b_i^\prime}{2\pi} \frac{M_X}{M_I}. \qquad (1)$$

In a combination δ [16] b_i can be eliminated keeping $b_i^\prime$, where,

$$\delta = 7\alpha_3^{-1}(m_Z) - 12\alpha_2^{-1}(m_Z) + 5\alpha_1^{-1}(m_Z). \qquad (2)$$

Eqn.(1) and Eqn.(2) together leads to,

$$\delta = \frac{1}{2\pi}(7b_3^\prime - 12b_2^\prime + 5b_1^\prime) \ln \frac{M_X}{M_I} \equiv \frac{\Delta}{2\pi} \ln \frac{M_X}{M_I}. (3)$$

We get $\delta = 0$ when $\alpha_3(m_Z) = 0.1144$ with $\alpha_1(m_Z) = 0.01696$ and $\alpha_2(m_Z) = 0.03371$. For our purposes the intermediate symmetry group is $G_I \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$. If we restrict ourselves to only those Higgs scalars which can arise from superstring models with Kac-Moody levels one or two [17], we can characterize the intermediate scale models by a set of five integers $(n_L, n_R, n_H, n_C, n_d)$. Here n_C refers to the number of (8,1,1,0), n_H means the number of (1,2,2,0) fields and n_L and n_R means the number of (1,2,1,1)+(1,2,1,-1) and (1,1,2,1)+(1,1,2,-1) fields under G_I and n_d refers to the number of pairs of Higgs doublets below the scale M_I where

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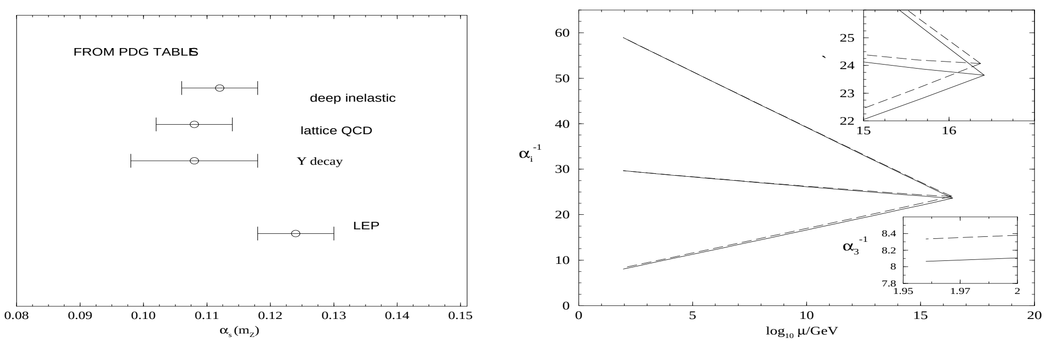

Figure 1. (a) Various measurements of $\alpha_3(m_Z)$ (b) The one-step unification case. The lower box shows the values of $\alpha_3(m_Z)$ required to achieve one-step unification.

the gauge symmetries are that of MSSM. Inserting the relevant beta function coefficients [7] and using $0.110 \leq \alpha_3(m_Z) \leq 0.130$ we get,

$$-10 \leq (-9 + 21n_c - 9n_H + 6n_R - 9n_L) \leq 6. \quad (4)$$

Table 1 catalogs these models in the decreasing order of minimality. In scenario IX the theory below the scale M_I has four Higgs doublets ($n_d = 2$).

Now we are in a position to proceed to a full two-loop analysis. The Yukawa couplings can be defined by the superpotential invariant under the intermediate symmetry. The variation of the unification scales with that of $\alpha_3(M_Z)$ have been plotted [7] in Figure (2a). The predictions of the intermediate scale have been plotted in Figure (2b) and that of the unification coupling $\alpha_G(M_X)$ have been plotted in Figure (2c) for various models. The grand desert case can be recovered from the meeting point of all the curves in Figure (2b) that is when the intermediate scale is equal to the GUT scale. In model VI the unification scale becomes low in the low α_3 region; for model V the same thing happens for high $\alpha_3(M_Z)$ region. As the dimension five proton decay can be suppressed in the SO(10) models by some additional mechanism [18], we plot the dominant dimension six decay mode in Figure (2d). Near the GUT scale the Yukawa couplings are large and they fall quickly below the GUT scale. The Yukawa couplings tend to pull the individual lines towards the low α_3 region wheres in the high α_3 models (like V and VII) the gauge interactions have exactly the reverse effect. This causes the curvature in the graphs near the GUT scale which is purely a two-loop effect. Using $m_\tau(m_\tau)$ GeV, we calculate the value of $\tan\beta$ at low energy. Once the value of $\tan\beta$ is known unique predictions for $m_t(m_t)$ and $m_b(m_b)$ follows. The pole mass [19] has been calculated from the running mass for each value of $\alpha_3(m_Z)$. The predictions of m_t^{pole} is plotted in Figure (3a). The value of $R_b(m_t) \equiv \frac{m_b(m_t)}{m_\tau(m_t)}$ are plotted in Figure (3c), and when the prediction of $m_b(m_t)$ is extrapolated to the mass scale of the bottom quark we get Figure (3d). The lower bounds on $\tan\beta(\sin\beta)$ follows directly from an upper bound on the top quark Yukawa coupling. These bounds have been plotted in Figure (3b). In our models the intermediate $B-L$ symmetry is broken by the Higgs scalars $16 + \overline{16}$ fields of SO(10). We will consider two different scenarios by which Majorana mass of the right handed neutrino can be generated. (a) Using a higher dimensional operator of the form $\frac{h}{M_X} 16_F 16_F 16_H 16_H$ written in terms of SO(10) representations. The subscripts F and H mean fermions and scalars respectively. When 16_H gets a VEV a large Majorana mass of the order $h\ v_R^2/M_X$ is generated. (b) Introduction of additional singlets to have a generalized see-saw mechanism [10,20]. In Figure (4a) and Figure (4b) we have plotted the left

Table 1
The minimal models which satisfy the condition in Eqn.(4). When the quantity Δ is positive (negative) the model gives rise to an intermediate scale for the lower (higher) values of $\alpha_3(m_Z)$ than the one step unification case for which $\alpha_3(m_Z) = 0.1144$ at the one-loop level. In the case $\Delta = 0$ the intermediate scale is unconstrained at the one-loop level.

Model	n_L	n_R	n_H	n_c	n_d	Δ
I	0	2	1	0	1	-6
II	0	3	1	0	1	0
III	0	4	1	0	1	6
IV	0	3	2	0	1	-9
V	0	4	2	0	1	-3
VI	0	5	2	0	1	3
VII	1	5	2	0	1	-6
VIII	1	1	1	1	1	0
IX	0	3	2	1	2	–

handed neutrino masses in scenarios (a) and (b) modulo the unknown Yukawa couplings h and h' in various models [see Ref[7]]as a function of $\alpha_3(m_Z)$.

We know that a tau neutrino mass of the order of a few electron volts is preferable if neutrino is to be a candidate for the Hot Dark Matter (HDM). We see that in scenario (a) a tau neutrino in the range of 1-10 eV can be achieved in all the models depending on the value of $\alpha_3(m_Z)$. On the other hand, scenario (b) can predict a tau neutrino mass in the 1-10 eV range for models V, II and VI.

To conclude, we have summarized a two-loop RGE analysis of the gauge and Yukawa couplings in a class os SUSY unified theories with intermediate scales. The presence of a color octet (model VIII) above the intermediate scale can push the unification scale to the *string* scale for α_3 in the range $0.118 - 0.119$. In this scenario the bottom quark mass prediction is attractive and the τ neutrino has a suitable mass to become a candidate of hot dark matter. The equality of the left and right handed gauge couplings also remain preserved above the intermediate scale in this scenario as $n_L = n_R$.

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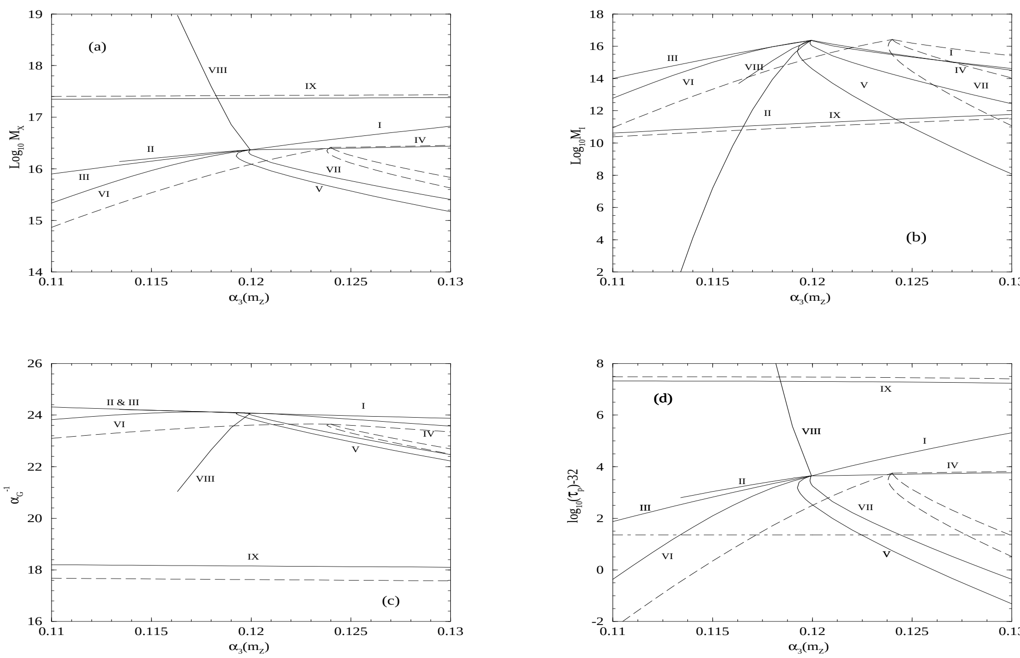

Figure 2. Predictions for (a) Unification scale, (b) Intermediate scale, (c) Unification gauge coupling, and (d) Proton life–time, for the models listed in Table I. Solid lines denote high $\tan\beta$ ($Y_1(M_X) = Y_2(M_X) = 1$; dashed lines denote the low $\tan\beta$ regime ($Y_1(M_X) = 1$, $Y_2(M_X) = 10^{-4}$). In Figure (d) the dotted line is the experimental limit $\tau_p = 5.5 \times 10^{32}\, yr$.

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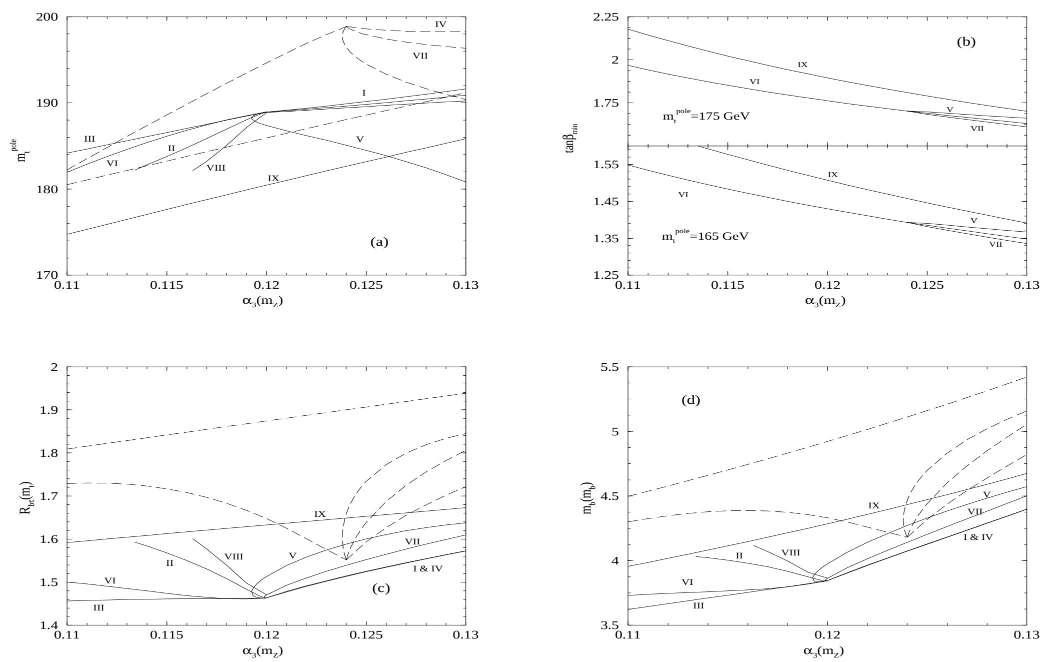

Figure 3. Predictions of (a) Pole mass of the top quark, (b) Lower bound on $\tan\beta$, (c) $R_{b\tau}$ at m_t, (d) Running mass of the b quark. Solid lines and dashed lines are as in Fig. (1).

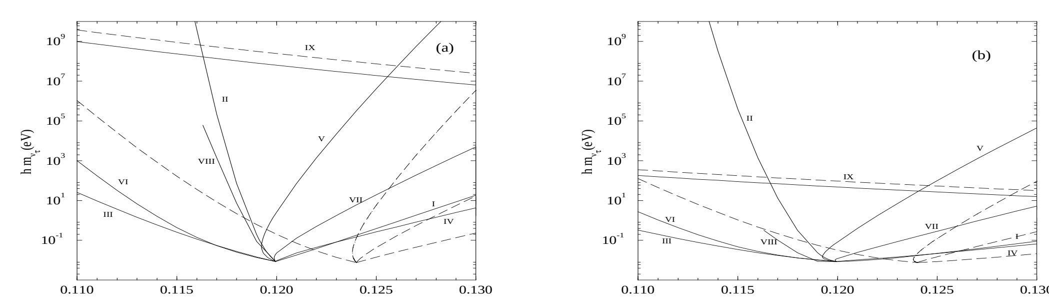

Figure 4. Predictions of the left–handed neutrino mass by see–saw mechanism by the two scenarios (a) and (b). Solid and dashed lines are as before.